ABOUT CERTAIN ASPECTS OF THE STUDY AND DISSEMINATION OF SHINICHI MOCHIZUKI'S IUT THEORY

IVAN FESENKO

This text aims to communicate in a compact form some factual information related to social aspects around Shinichi Mochizuki’s IUT theory\(^1\). Some more general issues, including mathematical ones, are discussed in two other papers\(^2\)\(^3\).

Number theory consists of many different areas and the distance from one area to the rest can be large\(^4\). To follow new fundamental developments one typically needs to study relevant prerequisites: for Deligne’s proof of GRH in positive characteristic one needs to know the relevant work of Grothendieck, for non-abelian linear developments in the Langlands program one needs to know linear representation theory, etc. The main prerequisite for IUT is the vast area of arithmetic geometry developed by Grothendieck and specialised in anabelian geometry developed in the last 30 years mostly in Japan. The distance from anabelian geometry to almost all other areas of number theory is substantial. Issues which have been preventing number theorists, especially in English speaking countries, from learning Grothendieck’s arithmetic geometry since the 1970s, and in particular understanding Deligne’s proof of GRH in positive characteristic, are well known.

Active experts in arithmetic anabelian geometry in 2012 outside Japan could be counted on the fingers of one hand. Thus, the expert perception of IUT could not involve as many mathematicians outside Japan as in the case of the theorems of Faltings or Wiles. Moreover, IUT is a rare pioneering development with many new concepts and ideas. To gain a good understanding of IUT, one has to invest an adequate large amount of time in a dedicated focused study of the theory starting with Grothendieck’s theory of étale fundamental groups and including several key developments of anabelian geometry. This cannot be done during a relatively short period of time.

To help mathematicians to study IUT, a substantial amount of time and effort have been dedicated since 2015 to the dissemination of IUT, via a 2-digit number of workshops and seminars, lectures and study groups. In particular, in 2018–2019 two year long IUT seminars at RIMS for new learners were conducted, and in 2021 four online international workshops on anabelian geometry, combinatorial anabelian geometry and IUT were conducted. A book\(^5\) by F. Kato, published in April 2019, presents various features of IUT to the wider audience. This book was in the list of top twenty bestselling books in all subject areas on amazon in Japan. It is awarded the Yaesu prize\(^6\). In view of the immense learning aspects, it was natural that those activities involved many young researchers and most of new experts in IUT are rather young.

The number of researchers who have mastered IUT is steadily growing. Learners of IUT have sent a 4-digit number of questions and remarks to the author, all addressed. Numerous surveys of IUT and a highly

---

\(^1\) The IUT papers are published in March 2021 by European Math Society https://www.ema-ph.org/journals/show_issue.php?issn=0034-5318&vol=57&iss=1. The IUT papers were made public in August 2012 and their files are available from its author page http://www.kurims.kyoto-u.ac.jp/~motizuki/papers-english.html. See those pages for various information on seminars and workshops on IUT. See also this page https://ivanfesenko.org/wp-content/uploads/2021/11/guidesiut.pdf for surveys of IUT, talks, workshops and seminars


\(^4\) this also depends on the stage of development of the area

\(^5\) https://twitter.com/FumiharuKato

\(^6\) https://twitter.com/yaesu_paseo/status/1190084381529886721?ref_src=twsrc%5Etfw
popular book on IUT, by mathematicians from several countries, presented the theory in different ways. There are more mathematicians able to produce professional reports on the IUT papers than the number of such reports on the main rare math breakthroughs at the time of their publication. No valid math evidence of any serious fault in IUT, confirmed by professionals, has been found by anyone. Minor oversights have been found and corrected. To this day there remains no mathematically substantive reason whatsoever to doubt the validity of IUT.

Some other researchers have tried to study IUT just for a short while, without mastering Grothendieck’s arithmetic geometry and anabelian geometry and/or without attending the workshops on IUT, and have failed. A tiny subset of such mathematicians were active in publicly making negative and absurd remarks about IUT, inevitably devoid of any serious math substance. Few American bloggers chose to spread fake news and disinformation about IUT. Their irresponsible actions might have affected others unable to make their own mind or to find experts to talk with. Even worse, their actions might have discouraged some bright young mathematicians to start or continue their hard intensive work on substantial advances on key open fundamental problems and theories.

1. On mathematical environment around IUT, briefly. Class field theory, the heart of algebraic number theory, has several important generalisations. They include linear non-commutative Langlands correspondences, non-linear non-commutative anabelian geometry and non-linear commutative higher class field theory. By various reasons the first generalisation\(^7\) has attracted many times more researchers than the second and the third, but all of these generalisations of class field theory are fundamentally important. Most of the central problems in the second and third generalisations of class field theory have been settled\(^8\).

The IUT theory would have been impossible without arithmetic anabelian geometry, including Mochizuki’s famous proofs of the Grothendieck conjecture and his absolute and mono-anabelian geometry. Anabelian geometry was started in works of Neukirch–Ikedà–Uchida–Iwasawa for small fields (such as number fields or their completions), and from a different motivation it was proposed by Grothendieck for hyperbolic curves over number fields. The main leading country in arithmetic anabelian geometry is Japan, and the first three contributors to anabelian geometry for hyperbolic curves over number fields were H. Nakamura, A. Tamagawa and Sh. Mochizuki.\(^9\) Below ‘anabelian geometry’ will mean ‘arithmetic anabelian geometry’. In the last thirty years a vast body of fundamentally important results in anabelian geometry were established. These developments were essentially left unnoticed in many countries and outside a small group of experts. Anabelian geometry is very much different from such other developments such as the Langlands correspondence or the classical Diophantine geometry and the expertise in those areas is not of much use in anabelian geometry.

The IUT theory uses some key theorems in anabelian geometry, as well as its later developments such as absolute anabelian geometry and mono-anabelian geometry. The total volume of relevant papers in anabelian geometry used in one or another extent in IUT is huge. One starting observation for arithmetic deformation theory, i.e. IUT, is that unlike the usual algebraic geometry in which working with schemes locally corresponds to working with commutative rings, working with certain anabelian objects corresponds to working with large nonabelian topological groups, using one operation instead of two, hence with new options to perform kinds of arithmetic deformation, not available in the standard arithmetic geometry. There is an associated fundamental problem to measure the deviation of certain diagrams of groups and maps between groups from being commutative. This problem is solved by IUT in the specific setting of hyperbolic

\(^{7}\) even though it is still lacking a version parallel to general class field theory, see the next footnote  
\(^{9}\) In the 1990s, a series of results about anabelian properties of Galois groups of global and higher global fields, i.e. birational anabelian geometry, were obtained by F. Pop. Since the early 1990s, F. Bogomolov suggested and developed, later in collaboration with Yu. Tschinkel, his birational anabelian geometry for varieties of dimension > 1 over algebraically closed fields, this theory is quite different from arithmetic anabelian geometry in many respects.
curves related to elliptic curves over number fields, thus eventually providing bounds on certain deformations which are then translated into the proof of abc inequalities.

2. **The study of IUT.** Links to various study materials about IUT are available from pages of the author of IUT\(^{10}\). The total amount of time dedicated to the verification process of IUT by mathematicians is several decades. It is likely to be the largest time ever spent in the history of mathematics on the verification of a mathematical work prior to its publication. Vast opportunities to study IUT have been open since September 2012. It is possible to contribute useful questions, comments, remarks, e.g. in relation to more conventional parts of the theory, e.g. such as those that came in 2012 from classical number theorists.

The IUT papers have been checked and verified \(^{11}\) by (a) a group of appointed referees, for 8 years, with 10 revisions of the original paper; (b) about 20 other mathematicians of many nationalities who have sent more than 1000 of their questions and remarks to the author, all answered and taken into account when relevant; (c) a RIMS seminar in 2015 and two international IUT workshops in 2015 and in 2016; (d) two RIMS seminars in 2018/2019 and in 2019/2020; (e) an international online seminar on IUT\(^{12}\) in 2020/2021 involving researchers from seven countries; (f) two online RIMS workshops in 2021.

Mathematicians accept a math work as correct either checking the work themselves or, much more often, by accepting without checking when the following conditions are satisfied: (a) the work has been published and hence well peer reviewed; (b) there have been conferences on which the work has been reported; (c) the author or other experts in the work have answered any questions he received about the work; (d) AMS mathscinet review was positive; (e) there are no published papers which claim that something is wrong in that work. All these conditions are satisfied or over-satisfied in the case of IUT. Moreover, one year after the publication of the IUT papers there are already further developments of IUT. Concerning (e) one can mention that there is a 2018 short report, but it is not a math paper with full proofs, see 3.3.

Specific features of the situation with IUT: (1) IUT is part of (arithmetic) anabelian geometry of hyperbolic curves over number fields which intensively uses Grothendieck’s vision and theories. The 50 years old Grothendieck’s heritage has not been digested by the overwhelming majority of number theorists. The number of experts in this subject area was very small in 2012; (2) in view of (1), to study the IUT papers requires an immense investment of time and effort. All new learners of IUT in the last few years are young researchers who have typically zero political weight to "convince" other mathematicians; (3) irresponsible, non-ethical behaviour of few mathematicians, see 3.1 and 3.3, who have been claiming something was wrong in IUT without providing a math paper with full proofs; (4) the increasing tendency among mathematicians (probably in all areas) to non-critically follow someone else’s blog opinion without ever checking that somebody’s expert status with respect to the subject of that person’s post; (5) it is not a tradition among Japanese mathematicians to run political campaigns to promote any math work.

One can occasionally hear a request to provide more details and explanations for the IUT papers, with an associated psychologically comfortable attitude to wait for this to happen.\(^{13}\) This request indicates lack of knowledge: there are already many surveys of IUT and numerous workshop talks about all of its aspects.

**Recommendations to mathematicians interested to study IUT.** Pathways to study IUT are available from many sources including www-links in footnotes of this text. If you find a piece of IUT looking to you as an error and you cannot resolve it, document your evidence and contact the author or its learners to discuss.

3. **On negative aspects of reaction to IUT.**

3.1. **On public reaction to IUT from few mathematicians.** Mochizuki’s work includes fundamental contributions in numerous directions: Hodge–Arakelov theory, anabelian geometry, mono-anabelian geometry, Grothendieck–Teichmüller group, \(p\)-adic Teichmüller theory,

\(^{10}\) [http://www.kurims.kyoto-u.ac.jp/~motizuki/top-english.html](http://www.kurims.kyoto-u.ac.jp/~motizuki/top-english.html)


\(^{12}\) [http://www.kurims.kyoto-u.ac.jp/~bcollas/IUT/IUT-schedule.html](http://www.kurims.kyoto-u.ac.jp/~bcollas/IUT/IUT-schedule.html)

\(^{13}\) ‘There is no royal road to geometry’, as Euclid remarked when Ptolemy complained his proofs were hard to follow.
inter-universal Teichmüller theory. Except for the last direction, none of his work has ever been criticised because it was read and appreciated by experts in the subject area.

There are no normal mathematical texts, with full proofs, about any faults in IUT. There are no published papers on faults in IUT, papers which have passed peer review.

Few mathematicians chose to publicly talk or comment in a benighted way about IUT and its study, while being fully aware about their lack of expertise in the subject area. They made public their ignorant negative opinions about a fundamental development in the subject area where they have empty research record, with no evidence of their serious study of it, and, crucially for professional mathematics, without providing any solid math evidence of errors in the theory.\(^\text{14}\)

Talking exclusively with non-experts, who have very weird ideas about IUT, can only produce weird outcomes. Unfortunately, non-expert negative opinions about IUT seeded a pernicious mistrust of this rare breakthrough and pioneering math research in general. Their behaviour contributes to the erosion of professional norms. For instance, there are no active US researchers in anabelian geometry of hyperbolic curves over number fields, but most of negative comments about IUT originated from a tiny subset of US mathematicians.

Recall that Article 6 of the European Math Society Code of Practice\(^\text{15}\) states, ‘Mathematicians should not make public claims of potential new theorems or the resolution of particular mathematical problems unless they are able to provide full details in a timely manner’. This article is also applicable to public statements that some important mathematical theories are mistaken. This EMS article is in full agreement with the fundamental principle of presumption of innocence and burden of proof. Any accusation that a paper/theory is incorrect must be supported by proof/evidence. In mathematics, proof/evidence means a mathematical paper with full details. In view of the importance of the IUT theory, such a paper should be submitted to a good journal (which arranges good quality of peer reviews) or should have already been published by a good journal.

Some people like to talk about some kind of controversy about the status of IUT. This is not an argument that can hope to be accepted: in order to have a controversy about a mathematical work there should be genuine experts on both sides of the argument able to provide valid math arguments which can pass peer review. This is plainly not the case for IUT: no internet critical remark is known which can become part of a paper to pass peer review and none has been published. There is only one professional side, the side of experts in IUT, which includes many those who have worked for years to learn the subject area and the theory. They, together with the referees and the group of editors processing the IUT papers, have all concluded that the IUT papers have no mathematical flaws. Part of this process was a truly unprecedented event when the author of IUT has kept investing a lot of time in answering a 4-digit number of questions since 2012.

Online aspects have negatively contributed to these damaging developments. Shallow posting is the only venue for non-expert public chats about IUT.

3.2. Some articles about IUT in mass media. IUT has attracted a high level of interest from mass media. Most experts in IUT decline to answer journalists questions, so then journalists contact mathematicians who are not experts in anabelian geometry or even laypersons with zero publication record in number theory. Some articles, even in Nature, cite opinions of non-experts only, mathematicians with zero track record in anabelian geometry. Experience in areas such as classical Diophantine geometry, algebraic geometry, modularity, Galois representations, the Langlands program, aspects of local number theory does not enable one to professionally comments on deep work in anabelian geometry and IUT.

One of easiest ways for journalists to write their articles is to fabricate the existence of solid opposite points of view but in the case of IUT the journalists often fail to appreciate that they mix experts opinions

\(^{14}\) In one case, a mathematician lacking any expertise in algebraic number theory attempted to block work on grant proposals including new work in anabelian geometry and IUT.

\(^{15}\) https://euromathsoc.org/code-of-practice
(all of which are positive) with ignorant opinions of non-specialists who did not want to be kept in the loop in relation to the study of IUT.

Recommendation to serious journalists. Before interviewing a mathematician about IUT, first ask several simple questions such as their knowledge of and expertise in anabelian geometry, attendance of conferences on anabelian geometry and IUT, the number of hours spent on the study of IUT, and whether they asked questions about IUT to the author of IUT or experts in IUT.

3.3. One example of ‘study’ of IUT. Failing to properly study the theory, some invented their own totally incorrect versions of IUT and claimed the identity of their caricature version with IUT, without providing mathematical evidence.

In 2013–2017 not a single concrete mathematical remark indicating any essential issue in IUT was produced. However, Scholze, who does not have work in anabelian geometry and has not participated in any anabelian geometry and IUT workshops, kept talking publicly about faults in IUT since 2014 without ever providing any math evidence. After a lot of pressure from several mathematicians, he visited RIMS, together with Stix, in March 2018, just for several days. Instead of explaining what kind of ‘mistakes’ they see in IUT, they were kindly given introductory lectures on IUT by experts.

The first report (not a paper with full proofs) about the meeting, shortly after the meeting and several months before the award of the Fields Medals, included a hugely incorrect version of IUT, based on a gross erroneous oversimplification of IUT. The report demonstrated serious problems with understanding of the theory, e.g. the difference between frobenius-like structures and étale-like structures in IUT. Their caricature version identifies non-freestanding isomorphic subobjects of the log-theta lattice! Of course no proof of the equality of this version and the original IUT theory was given. The report includes such phrases as ‘we are certain that even with all subtleties restored, the issue we are pointing out will prevail’, ‘it seems to us’. The report essentially denies the use of anabelian geometry and infinitely many theatres in IUT. For various details see this page and these report and text of the author of IUT.

The German mathematicians intended to make their report available online, however, after reading the comprehensive report of the author of IUT on their report and these comments, they changed their mind and abandoned plans to post their own report at that time. In his comprehensive report on their report the author of IUT formulated few questions to the German mathematicians which may have helped them to appreciate their mistakes. However, the second version of their report failed to address those questions. Moreover, it included new incorrect statements demonstrating lack of basic knowledge of more classical areas such as height theory and one of the Faltings papers.

Scholze unilaterally withdrew from any further correspondence or study of IUT since October 2018. His recent short zbl review of the IUT papers is additional evidence of his ‘study’ of IUT. It includes new mathematically incorrect statements such as statements about Hodge theatres and demonstrates sheer lack of understanding of the main concepts and structures of IUT.

It is not unusual to make a mistake in one’s mathematical study, but to publicly talk about faults in another theory for many years without ever having any valid evidence and refusing to discuss for 4 years is irresponsible. He has been directly violating Article 6 of the EMS Code of Practice for 8 years.

---

16 I wrote to him several times requesting to tell precisely what were the faults in IUT and discuss with experts. The author of IUT had invited Scholze to discuss any issues but Scholze did not follow.
17 For a popular presentation to high school students of the importance to use infinitely many theatres in IUT, one can watch F. Kato’s talk https://www.youtube.com/watch?v=fNS7H04DLAq&vl=en
18 https://www.kurims.kyoto-u.ac.jp/~motizuki/IUTch-discussions-2018-03.html
20 referred to in footnote 19, see especially its §2 and §5 of the report
22 see also Remarks 3.11.1 and 3.12.2 of IUT-III