New interactions between arithmetic geometry and quantum theory and neural networks

Ivan Fesenko

Summary

Two areas initiated by Grothendieck: topos theory and anabelian geometry, already have and will have many more interaction and applications with quantum theory and AI.

There is a great potential for the use of some concepts and visions of these math areas in quantum theory, quantum computing, and for understanding of deep neural networks and AI systems.

Expectations of intensive interaction of number theory and quantum theory

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However, deep facts of number theory play no role in questions of quantum mechanics...

I predict that in the next century we will witness deep applications of number theory in fundamental physics ...

I would think that quantum mechanics will be completely reformulated and that number theory will play a key role in this reformulation'

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Problems with foundations of quantum theory

Quantum theory has enormous conceptual problems in its standard formulation that used 100–70 years old mathematics.

These problems are typically ignored by modern physicists.

The biggest mistake is actually quite simple and shared by most people (including Bohr and Heisenberg), who believe that the "collapse" phenomenon has to do with a special physical process called "measurement", assumed to involve (self-)conscious beings.

However, the "collapse" of the wave function happens all the time, e.g. during radioactive decay of single nuclei, without the need for any macroscopic object, be it conscious or not.

The only alternative is the "many-worlds" interpretation, but it is itself inconsistent with the Born rule.

Problems with foundations of quantum theory

Classical theory (quantities are real valued) $\longrightarrow$ a quantisation of it,

but why should quantum quantities be real valued?

Why should quantum probabilities be in $[0, 1] \subset \mathbb{R}$?

Standard mathematics description of quantum theory assumes certain properties of space and/or time but the Planck scale hints otherwise!

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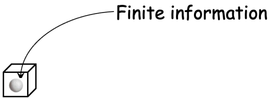
Real numbers and physics

Gisin: 'Real non-rational numbers have no periodicity in their infinitely many decimal digits, but

Finite volume $\Rightarrow$ finite information

- A finite volume of space can not contain infinitely many bits of information.
- Hence, the position of a classical particle is not a real number.



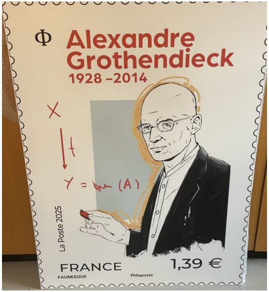
Mathematical real numbers are not Physically real

real numbers are not really real

Categories

In the 1940s S. Eilenberg and S. Mac Lane developed category theory to provide clearer structural approach to algebraic topology and to build bridges between algebra and topology.

Since the mid 1950s A. Grothendieck further developed category theory, topos theory and their applications in numerous directions, and most importantly in arithmetic geometry.



Toposes

The notion of topos was introduced in the early sixties by Grothendieck with the original first aim of bringing a topological or geometric intuition in parts of number theory where actual topological spaces do not occur.

Grothendieck invented topos theory to provide a mathematical underpinning for the missing étale site and étale cohomology theory needed in arithmetic geometry.

He realised:

many important properties of topological spaces X can be naturally formulated as properties of the categories $\mathrm{Sh}(X)$ of sheaves of sets on the spaces.

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Toposes

Toposes form a special subclass of categories, which is very suitable for the study of geometric, logical, syntactic, and semantic aspects of the relevant theory.

A topos is the most universal generalisation of a topological space.

At the same time, most geometric intuitions and constructions can be transposed to the new notion of topos.

Similar to a quantisation of a classical physical theory, constructions in topos theory can often be understood by looking at them in the category of sets or geometrical categories first, and then lifting to the general case.

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Toposes

Grothendieck introduced an abstract notion of covering replacing the topological space X by

a *site* (C, J) consisting of a (small) category C and a Grothendieck's topology (a generalized notion of covering) J on it.

A Grothendieck topos is any category equivalent to the category of sheaves on a site.

A topos has various features similar to the category of sets.

However, unlike sets, the law of excluded middle does not need to hold in a topos.

Statements about a topos are not necessarily either true or false, they can be true somewhere and false somewhere.

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Toposes

The crucial unifying notion of topos is to provide the common geometric intuition for many areas of mathematics and to connect continuous with discrete.

$X \to \mathrm{Sh}(X)$ is an embedding of continuous topological space X into a category $\mathrm{Sh}(X)$ which is a discrete structure.

Grothendieck:

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Examples:

Topos of sheaves on a topological space (in the first approximation, think of functions on open subsets, appropriately glued), for example:

- the sheaf of regular functions on a variety
- the sheaf of differentiable functions on a differentiable manifold
- the sheaf of holomorphic functions on a complex manifold
- the sheaf of continuous real-valued functions on any topological space

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Topos theoretical approach to quantum theory

Using topos theory leads to a reformulation of quantum theory which in several aspects looks like classical physics,

propositions can be given truth values without using concepts of measurement or external observer,

and the logic can be non-Boolean

A topos has an internal logical structure that is similar to the way in which Boolean algebra arises in set theory, but instead of two truth values 1 and 0, goes outside Boolean logic with truth values are in a larger set.

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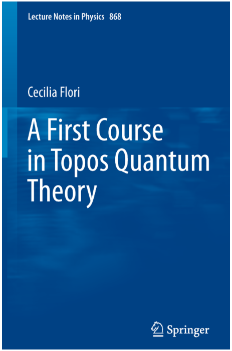
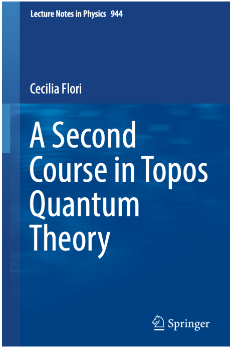
Isham–Döring's topos theoretical approach to quantum theory

builds locally on (the topos of presheaves of) commutative (hence classical) sub-algebras of the algebra of all bounded operators on the quantum theory's Hilbert space.





Classifying topos

According to topos theory,
each first-order geometric theory has a classifying topos such that
for any other topos, the category of geometrical functors from that other topos to the classifying topos is equivalent to the category of models of the theory in that other topos.

There are different ways of looking at the classifying topos of a theory: logic, geometric, semantic, and syntactic.

Relationships between them naturally arise from invariants of the classifying topos in terms of different sites of definition for it.

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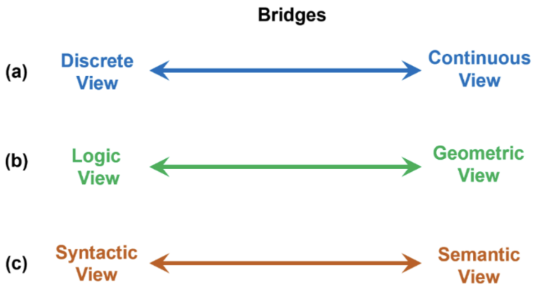
Relating different aspects via classifying toposes

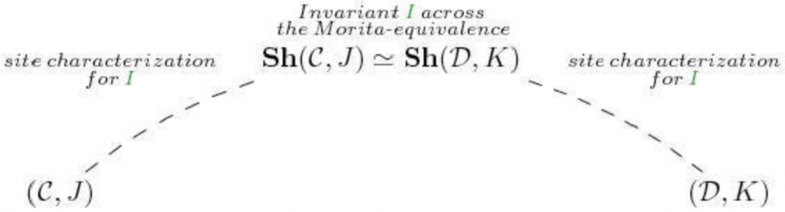


Bridges in topos theory

Caramello's theory of topos-theoretic 'bridges', which is a general theory of relations between the contents of different mathematical theories.

A Grothendieck topos can be the classifying topos of many geometric logic theories which are Morita-equivalent with each other.

By this transformation, we can not only bridge between logic and geometry but also extract the semantic information that is stable under the changes of syntactic presentation.



Topos theory and DNN

Classifying toposes and their invariants are at the heart of recent developments in applications of topos theory to deep neural networks.

Research in this direction has been conducted by L. Lafforgue, O. Caramello, J.-C. Belfiore, D. Bennequin.

To an artificial deep neural network (DNN) one can associate an object in a canonical Grothendieck's topos; its learning dynamic corresponds to a flow of morphisms in this topos.

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There are many invariants of toposes, e.g. compact, two-valued, locally connected.

Studying invariants of classifying toposes appropriate of DNN leads to new important links between their syntactic and semantic aspects and a better structural understanding of key issues of the models.

In turn, this can lead to a large range of applications in AI and 6G.

Topos theory and DNN

The theory of sites (C, J), where C is a category and J is a Grothendieck topology on it, simultaneously specialises in the discrete category C and the continuous topology J on it.

The classifying topos can be viewed as the completion of a category C with respect to its Grothendieck topology J.

Topos theory assumes that a (Grothendieck) topology J is available for category C.

However, we are often given unstructured data with an unknown structure or topology.

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Topos theory and AI: from discrete to continuous

Current deep representation learning methods (contrast learning, mask learning, and deep manifold learning) are based on different underlying assumptions and lack theory for a unified description.

The classifying topos may help to reveal the methodological commonalities of various representation learning methods, and based on this, to design new methods that can enhance representation learning.

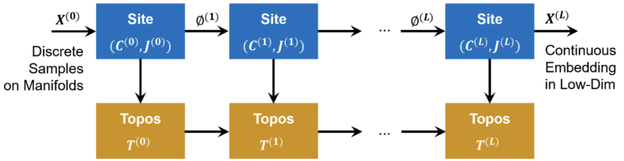

Figure 3. Learning topos representations: from discrete samples to continuous embedding using deep manifold transformation. This links ***from discrete spaces to continuous spaces***.

Topos theory and DNN

Topos theory provides powerful tools to tackle the challenges of the mathematical foundations of AI.

Topos-theoretic representations can help to build more efficient, reliable, and interpretable deep learning models.

Topos theory has the potential to unify multiple paradigms and techniques in AI, facilitating a more systematic and unified way of developing novel AI techniques.

Toposes and étale objects in arithmetic geometry

The notion of a geometric morphism in topos theory has allowed to build general cohomology theories which cannot be otherwise produced.

The Grothendieck definition of étale sites, étale fundamental group and étale cohomology uses toposes.

For any geometrically integral (quasi-compact) scheme X over a perfect field k one has its étale fundamental group $\pi_1(X)$.

Example. If C is a complex irreducible smooth projective curve minus a finite set of its points, over an algebraically closed field of characteristic 0,

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Anabelian geometry

A *hyperbolic curve* C over a field k of characteristic zero is a smooth projective geometrically connected curve of genus g minus r points such that the Euler characteristic $2-2g-r$ is negative.

The étale fundamental group of a hyperbolic curve is highly nonabelian, its centre is trivial.

Question 1 (Grothendieck). Are hyperbolic curves over number fields anabelian, i.e. can one restore the curve from its étale fundamental group?

A partial case of Q1 was positively answered by Nakamura and Tamagawa and then in full generality by Mochizuki.

So, unlike the general case of affine smooth varieties over fields which are determined by their *ring* (two operations) of functions, associated to polynomial equations defining the variety, anabelian curves over number fields are determined by their *group* (one operation) π_1.

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A point x in $X(k)$, i.e. a morphism $\mathrm{Spec}(k) \longrightarrow X$, determines, in a functorial way, a continuous section $G_k \longrightarrow \pi_1(X)$ (well-defined up to composition with an inner automorphism) of the surjective map $\pi_1(X) \longrightarrow G_k$.

Question 2 (Grothendieck). Section conjecture. For a geometrically connected smooth projective curve X over k, of genus > 1, is the map from rational points $X(k)$ to the set of conjugacy classes of sections

$$x \mapsto D_x = \mathrm{Stab}(x)$$

surjective? (injectivity was already known).

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Stabiliser formalism in quantum computing

One of the key issues for quantum algorithms is whether they can run in *polynomial time*, instead of *exponential time*. Controlling loss of information/error correction is crucial.

In quantum error correction one uses stabiliser groups in finite dimensional complex spaces.

A map

$$s \mapsto D_s$$

from quantum states s in a $2n$-dimensional vector space over $\mathbb{C}$ to their stabiliser group D_s (unitary matrices acting trivially on s) is *injective*.

However, D_s has too many generators (about 4^n).

Calderbank–Rains–Shor–Sloane, Gottesman, Aaronson–Gottesman (2008) considered the intersection $D_s \cap P_n$.

Here P_n is the group of n-qubit Pauli operators: all tensor products of n Pauli matrices

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Quantum computing

It is easy to show that on the subset of quantum states that are stabilised by exactly 2^n elements of P_n the map

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is still *injective*.

This subset is further characterised as obtained from $|0\rangle^{\otimes n}$ by CNOT, Hadamard, phase gates only.

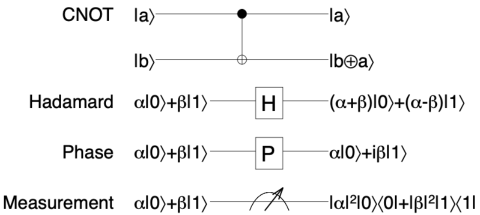

FIG. 1: The four types of gate allowed in the stabilizer formalism

Injectivity of section map

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Arithmetic geometry and quantum computing

The variety of decomposition groups and absolute Galois groups in number theory is much larger than just the Clifford group (the group of unitary matrices that normalise P_n) and P_n in quantum computing.

One perspective is to investigate whether analogues of the decomposition groups and absolute Galois groups in arithmetic geometry will provide new classes of groups useful for quantum computing.

This may allow to go beyond the Clifford ground in quantum computing, an important open challenge in quantum computing.

Anabelian geometry and IUT

Algebraic geometry involves locally the correspondence between affine varieties and commutative rings.

Anabelian geometry for hyperbolic curves over number fields and other fields is a correspondence between these geometric objects and their arithmetic fundamental groups (or slightly more complicated objects).

Fundamental groups are highly non-commutative, but they have one algebraic operation, not two.

This opens the perspective to try to perform deformations of these geometric objects not seen by algebraic geometry, using the fact that there are more maps, group homomorphisms and variations of those between topological groups in comparison to morphisms between commutative rings.

However, important associated diagrams are not commutative.

The main contribution of Mochizuki in the IUT theory for certain hyperbolic curves (e.g. an elliptic curve minus a point) is a new fundamental understanding of how to bound from above the deviation from commutativity of certain crucial diagrammes.

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IUT theory

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IUT works with deformations of multiplication which are not compatible with ring structure.

Deformations are coded in theta-links between theatres which are certain systems of categories associated to an elliptic curve over a number field.

Ring structures do not pass through theta-links.

Galois and fundamental groups (groups of symmetries of rings) do pass.

To restore certain rings from some groups that pass through a the theta-link one uses anabelian geometry results about number fields and hyperbolic curves over them.

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Deformations are coded in theta-links between theatres which are certain systems of categories associated to an elliptic curve over a number field.

Ring structures do not pass through theta-links.

Galois and fundamental groups (groups of symmetries of rings) do pass.

To restore certain rings from some groups that pass through a the theta-link one uses anabelian geometry results about number fields and hyperbolic curves over them.

Intriguing similarities between IUT and quantum theory

1. In IUT, number 6 plays important role in several respects.

In IUT, multiplication and addition, related to geometric and arithmetic symmetries, play a central role.

These two dimensions are reminiscent of the two parameters, one of which is related to electricity, the other to magnetism, employed in the study of layers of hexagonal lattices of graphene.

2. In mono-anabelian geometry and IUT one algorithmically reconstructs objects from étale fundamental groups.

IUT produces upper bounds on change of relevant data passing through the theta-link, using the action of étale fundamental groups which pass through the link unaffected.

Using appropriate group action on a flow of information, to control its loss, is useful in quantum computing.

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Intriguing similarities between IUT and quantum theory

3. IUT works with two types of topological monoid/group/ring structures: étale-like (coming from groups of symmetries) and frobenius-like (coming from 'ordered' objects) and then use various interactions and connections between these two structures. One of such connections is given by a generalised Kummer map.

These two structures are analogues waves and particles in quantum mechanics.

Interaction of frobenius-like and étale-like structures via the Kummer map in IUT may be sometimes viewed as analogous to the relation between particles and waves in quantum mechanics.

4. In IUT one has to go through a full closed loop before returning to original universe and passing to a set-theoretic log-volume.

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